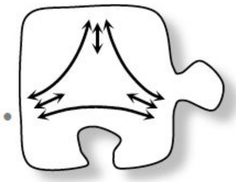
3.1.1 Are they the same?



Using the Multiplicative Identity

In this lesson, you will analyze a powerful tool for finding equivalent fractions. This way of thinking about equivalence can be applied to many different situations. You will see how it can be used to solve problems and to investigate new applications.

3-1. Ms. Vazquez has a reputation for telling math jokes. She started class today with this one:
Ron was picking up his pizza at the takeout window. The clerk asked him, "*Do you want your pizza cut into eight slices or twelve?*"
"*You'd better cut it into eight slices,*" Ron replied. "*I'm not hungry enough to eat twelve.*"
Some students thought the joke was funny. Do you? What is the fraction concept that makes the joke work?

3-2. LESS IS MORE
Frankie's mother made a big pan of lasagna last night. She cut the lasagna into fourths and ate one. Then she put away the three pieces of leftover lasagna by cutting them each in half and repackaging the results. This process fascinates Frankie. *"How can you make something smaller and still end up with more?"* she asked.

Write a fraction representing how much lasagna was left after Frankie's mother ate her dinner. Write another fraction representing how much lasagna was left after it was repackaged. Does she really have more lasagna after repackaging? Use what you know about fractions to explain.

3-3. ONE-DERFUL ONE
Frankie was thinking about her mother's portion control (cutting up the lasagna), when she noticed a very useful connection. She recognized that *any* fraction in which the numerator and denominator are the same is equivalent to 1. In addition, multiplying a number by 1 leaves that number unchanged.

$$\frac{3}{5} \cdot \boxed{\frac{2}{2}} = \frac{6}{10}$$

"WOW," she said*, "I can use this idea to find a whole bunch of equivalent fractions!"* She showed her team the work at right. *"Look!"* she said,*"It's the same as using a GIANT ONE!"*

a. Discuss Frankie's work with your team. Does it make sense? Is $\frac{3}{5}$ equivalent to $\frac{6}{10}$? How can you be sure?

b. Find at least two other fractions or ratios that are equivalent to $\frac{3}{5}$.

c. What Frankie calls a Giant One is more formally called the **multiplicative identity**. Discuss with your team why you think it is called that. Keep this name in mind as you finish today's lesson and be ready to share your ideas.

3-4. Use the idea of the Giant One to find at least four fractions that are equivalent to $\frac{9}{8}$.

3-5. SO MANY CHOICES

Bertrand was feeling confused. *"There are so many ways to write the Giant One! How do I know which one to use?"* he whined. How can Bertrand decide which Giant One to use? Work with your team to answer the following questions and come up with a strategy.

a. Find the missing numbers in the fractions below.

i. $\frac{3}{4} \cdot \boxed{\frac{\ }{\ }} = \frac{\ }{44}$

ii. $\frac{7}{12} \cdot \boxed{\frac{\ }{\ }} = \frac{\ }{60}$

iii. $\frac{18}{72} = \boxed{\frac{\ }{\ }} \cdot \frac{3}{\ }$

b. What computation could help you find the numbers to use in the Giant Ones?

3-6. Use what you have discovered about finding the necessary Giant Ones to complete the following problem.

$$\frac{35}{50} \cdot \boxed{\frac{\ }{\ }} = \frac{\ }{10}$$

a. How is the Giant One that you used here different from the ones that you found in problem 3-5?
b. Can you think of a different way to make sense of this problem?

3-7. Sometimes it is useful to express a fraction in **lowest terms**. If you write a fraction in lowest terms, you use the smallest whole numbers possible to express the fraction. For example, $\frac{60}{70}$, $\frac{30}{35}$, and $\frac{6}{7}$ are equivalent fractions, but only the fraction $\frac{6}{7}$ is expressed in lowest terms. **Simplifying** a fraction is the process of rewriting it in lowest terms.

With your team, consider how the Giant One could help you simplify fractions as you answer the questions below.

a. Tessa has written the work shown below. Copy her work on your own paper. Then show the Giant One and each of the two equivalent fractions.

b. $$\frac{55}{500} = \frac{5 \cdot 11}{5 \cdot 100} = \frac{11}{100}$$

c. Does Tessa's work make sense? Is $\frac{11}{100}$ expressed in lowest terms? How can you tell? Be prepared to explain your ideas to the class.

d. Tessa is doing well! She decided to try another problem and wrote the work shown below.

e. $\frac{28}{60}=\frac{2\cdot 14}{2\cdot 30}=\frac{14}{30}$

f. Is her work correct? Is her fraction expressed in lowest terms? If so, explain how you can tell. If not, help her figure out the lowest terms for this fraction.

g. Work with your team to simplify each of the following fractions and write them in lowest terms.

i. $\frac{24}{36}$

ii. $\frac{30}{48}$

iii. $\frac{56}{98}$

3-8. Tessa has a brilliant idea. *"Look!"* she exclaimed. *"If I find the largest number that is a factor of both the numerator and denominator, I can simplify all the way in just one step!"* She wrote down the example below.

$$\frac{60}{72}=\frac{2\cdot 2\cdot 3\cdot 5}{2\cdot 2\cdot 2\cdot 3\cdot 3}=\frac{12\cdot 5}{12\cdot 6}=\frac{5}{6}$$

a. Remember that the largest factor of two numbers is called their **greatest common factor**. How did Tessa figure out that 12 is the greatest common factor of 60 and 72?

b. Factoring a number into its **prime factors** is to find the prime numbers that are its smallest factors. How can factoring into prime factors help you find the greatest common factor of any two numbers? Discuss this with your team and write down your ideas. Be prepared to share your ideas with the class.

c. Work with your team to use Tessa's idea to simplify each of these fractions in one step. State the greatest common factor for each part.

i. $\frac{24}{30}$

ii. $\frac{18}{45}$

iii. $\frac{30}{63}$

3-9. Additional Challenge: Andy and Bill found a quick way to figure out that $\frac{1}{3}\cdot\frac{3}{8}=\frac{1}{8}$. Show how the Giant One can help explain their shortcut. Then use the Giant One to calculate each product below quickly.

a. $\frac{2}{5}\cdot\frac{5}{7}$

b. $\frac{1}{2}\cdot\frac{3}{5}\cdot\frac{2}{3}$

3-10. Additional Challenge: Consider what you know about multiplying fractions and the Giant One as you solve the problems in parts (a) through (c) below.

a. Find two fractions between 0 and 1 that have a product of $\frac{1}{7}$.

b. Find two fractions between $\frac{1}{2}$ and 1 that have a product of $\frac{3}{4}$.

c. Find each of the products shown below. Then predict the next two problems in the sequence and find the products.

d. $\frac{1}{2} \cdot \frac{2}{3} =$

e. $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} =$

f. $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} =$

3-11. LEARNING LOG
Write a Learning Log entry to summarize what you learned today about the Giant One and its uses. Include examples of how the Giant One is used. Title this entry “The Giant One and Equivalent Fractions” and label it with today’s date.